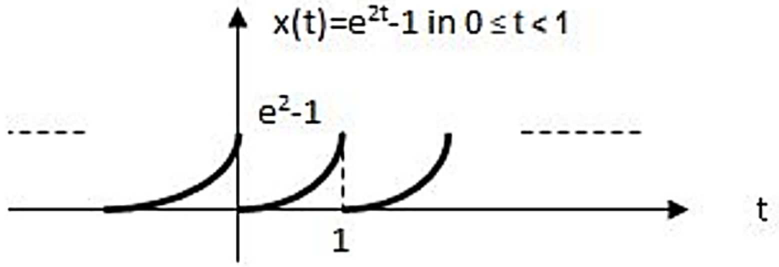
1- Given the following periodic signal:



A) Calculate the Fourier series coefficients C_k for the given signal **mathematically** **then write down the Fourier series expansion $x(t)$**

$$T = 1$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi$$

$$C_0 = \frac{1}{1}\int_0^1 (e^{2t} - 1)dt$$

$$C_0 = \frac{1}{2}[e^{2t} - t]_0^1 = \frac{e^2}{2} - \frac{1}{2} - 1 = \frac{e^2}{2} - \frac{3}{2}$$

$$C_k = \frac{1}{1}\int_0^1 (e^{2t} - 1)e^{-jk2\pi t}dt$$

$$C_k = \frac{1}{1}\int_0^1 (e^{(2-j2k\pi)t} - e^{-jk2\pi t})dt$$

$$C_k = \frac{1}{1}\int_0^1 (e^{(2-j2k\pi)t} - e^{-jk2\pi t})dt$$

$$C_k = \left[\frac{e^{(2-j2k\pi)t}}{2 - j2k\pi} + \frac{e^{-jk2\pi t}}{jk2\pi}\right]_0^1$$

$$C_k = \frac{e^{(2-j2k\pi)} - 1}{2 - j2k\pi} + \frac{e^{-jk2\pi} - 1}{jk2\pi}$$

Now, x(t) is given as:

$$x(t) = \frac{e^2 - 3}{2} + \sum_{k=1}^{\infty}\left(\frac{e^{(2-j2k\pi)} - 1}{2 - j2k\pi} + \frac{e^{-jk2\pi} - 1}{jk2\pi}\right)e^{jk2\pi t}$$

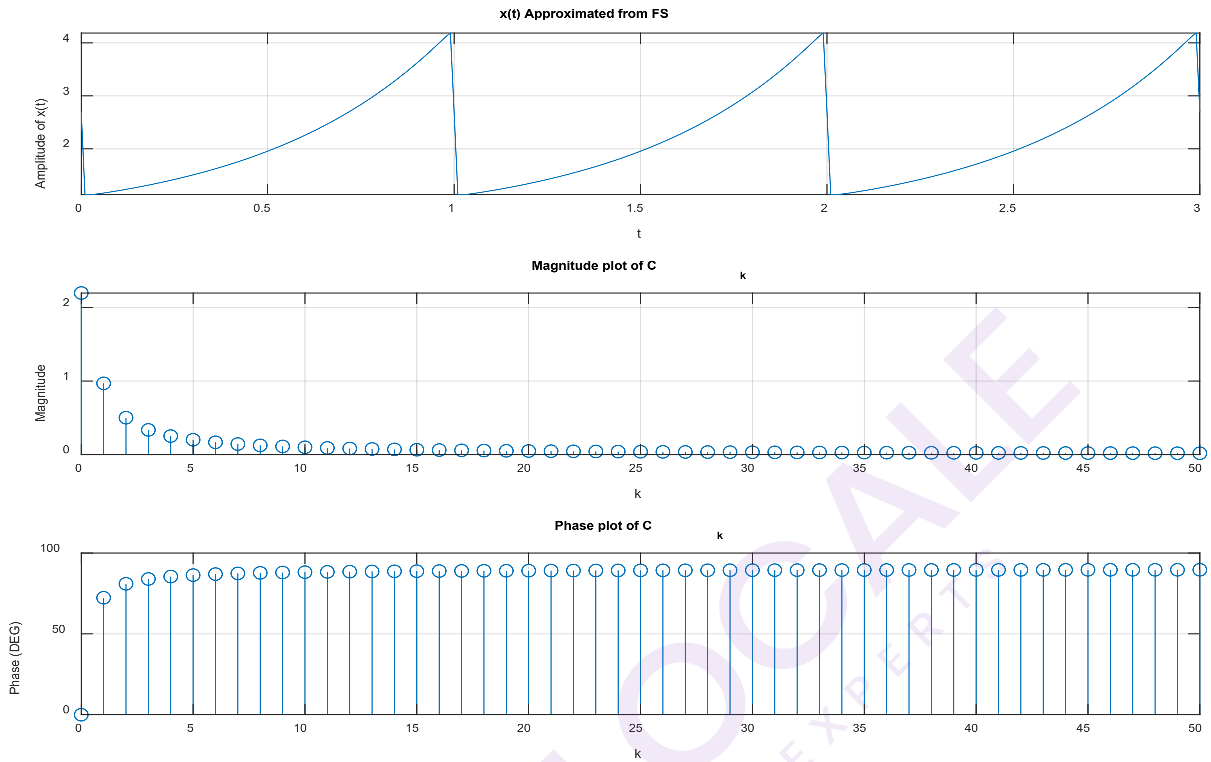
B) Verify your solution using Matlab by showing the magnitude and phase plots.

```
clc
```

```
clear all
close all
t = 0:0.01:3;
C_0=(exp(2)-3)/2;
x=C_0;
fs = 0;
for k=1:500
 C_k=((exp((2-j*2*k*pi) )-1)./(2-j*2*k*pi)+(exp(-1j*k*2*pi)-
1)./(j*k*2*pi));
 fs(k)=C_k;
x=x+C_k.* exp(j*k*2*pi*t);
end
subplot(311)
plot(t,x)
grid on
title('x(t) Approximated from FS')
ylabel('Amplitude of x(t)')
xlabel('t')

fs = [C_0 fs];
FS=fs;
k=0:50;
subplot(312)
stem(k,abs(FS(1:51)))
grid on
title('Magnitude plot of C_k')
ylabel('Magnitude')
xlabel('k')

subplot(313)
stem(k,angle(FS(1:51))*180/pi)
grid on
title('Phase plot of C_k')
ylabel('Phase (DEG)')
xlabel('k')
```



2- Given the following periodic signal with a period T:

3- $$f(t) = 2j - 4\sin(100\pi t - 40) - 10\cos(60\pi t + \pi - 70)$$

a) Find the period T and the fundamental frequency f_o

$$T_1 = \frac{2\pi}{100\pi} = \frac{1}{50}$$

$$T_2 = \frac{2\pi}{60\pi} = \frac{1}{30}$$

$$\frac{T_1}{T_2} = \frac{30}{50} = \frac{3}{5}$$

$$T_0 = 3T_2 = 3 * \frac{1}{30} = \frac{\mathbf{1}}{\mathbf{10}}\boldsymbol{s}$$

$$\boldsymbol{f_0} = \frac{\mathbf{1}}{\boldsymbol{T_0}} = \mathbf{10}\boldsymbol{Hz}$$
$$\boldsymbol{\omega_0} = \mathbf{2}\boldsymbol{\pi f_0} = \mathbf{20}\boldsymbol{\pi}$$

Solve the problem mathematically to find the exponential Fourier series then:

Expressing f(t) using Euler's formula:

$$f(t) = 2j - \frac{2}{j} * \left(e^{j(100\pi t-40)} - e^{-j(100\pi t-40)}\right) - 5\left(e^{j(60\pi t+\pi-70)} + e^{-j(60\pi t+\pi-70)}\right)$$

$$f(t) = 2j - \frac{2}{j} * \left(e^{j(5*20\pi t-40)} - e^{-j(5*20\pi t-40)}\right) - 5\left(e^{j(3*20\pi t+\pi-70)} + e^{-j(3*20\pi t+\pi-70)}\right)$$

Now, it can be deducted from the f(t) expression:

$$C_0 = 2j$$

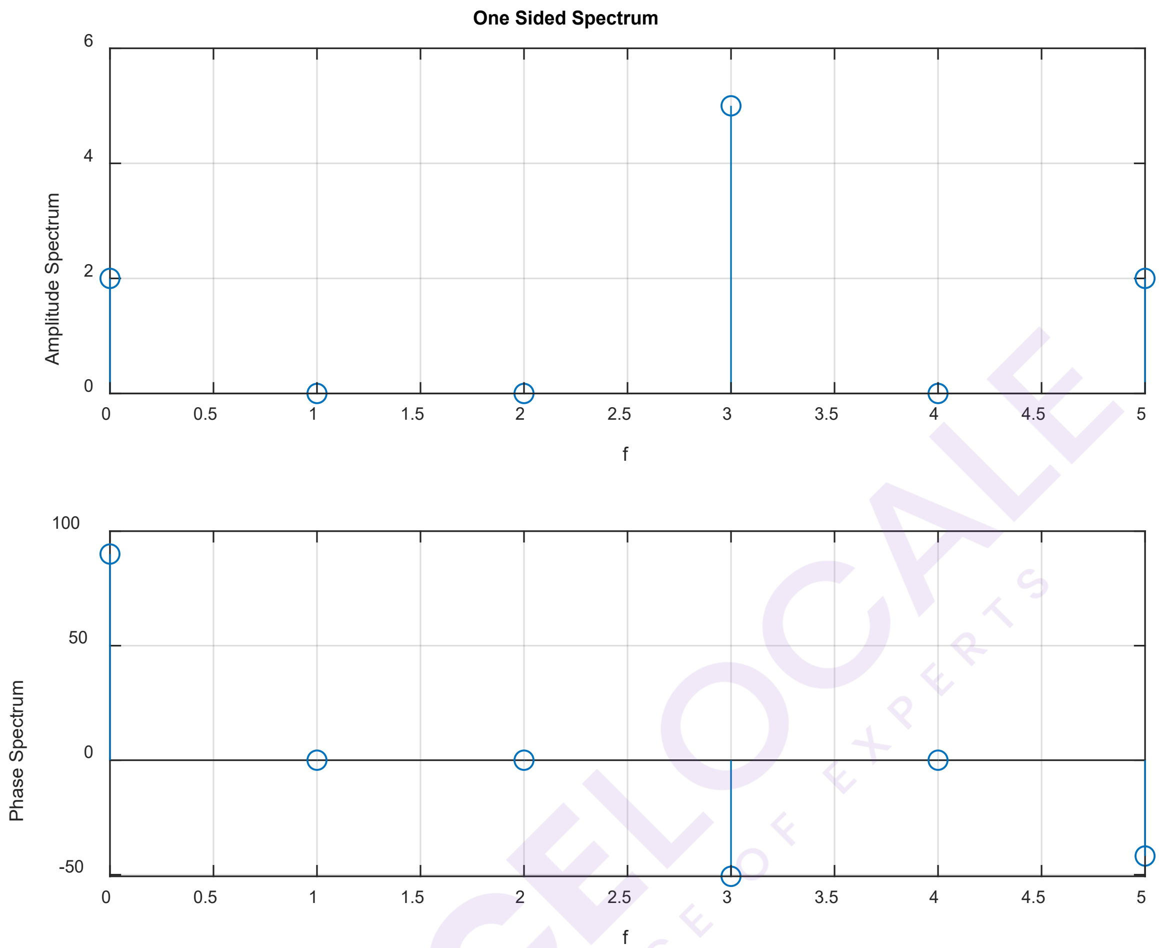
$$C_3 = -5e^{j(\pi-70)}; \quad C_{-3} = -5e^{-j(\pi-70)}$$

$$C_5 = -\frac{2}{j}e^{j(-40)}; \quad C_{-5} = -\frac{2}{j}e^{-j(-40)}$$

b) Use Matlab to plot the one sided spectrum.

```
clc
clear all
close all
C_k = zeros(1,6);
C_k(1) =2*1j;
C_k(4) = -5*exp(1j*(pi-70));
C_k(6) = (-2/1j)*exp(1j*(-40));
f=0:5;
subplot(211)
stem(f,abs(C_k))
grid on
xlabel('f')
ylabel('Amplitude Spectrum')
title('One Sided Spectrum')

subplot(212)
stem(f,angle(C_k)*180/pi)
grid on
xlabel('f')
ylabel('Phase Spectrum')
```



c) Use Matlab to plot the double sided spectrum.

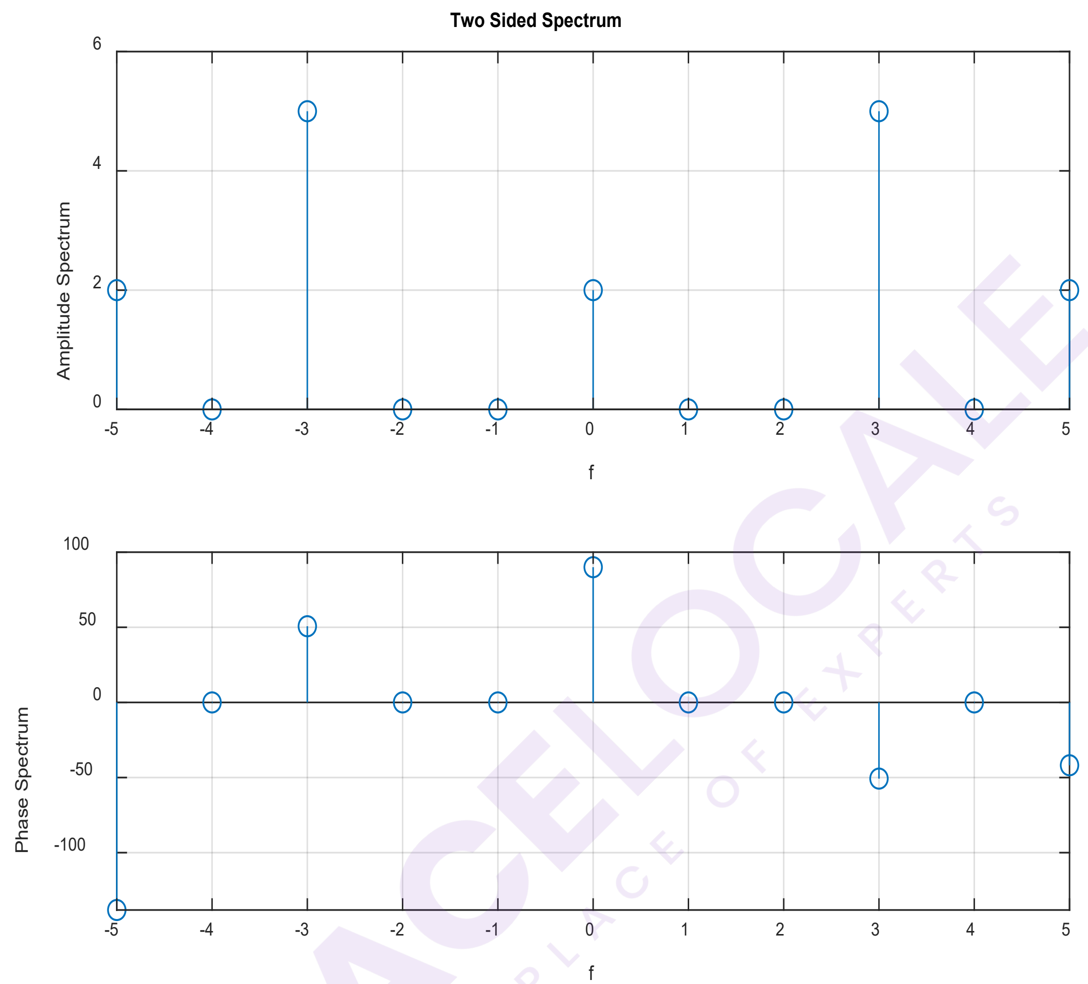
```
clc
clear all
close all
C_k = zeros(1,11);

C_k(1)  = (-2/1j)*exp(1j*(40)); %C-5
C_k(3)  = -5*exp(-1j*(pi-70)); %C-3
C_k(6)  =2*1j; %C_0
C_k(9)  =-5*exp(1j*(pi-70)) ; %C_3
C_k(11)  = (-2/1j)*exp(1j*(-40)); %C_5

f=-5:5;
subplot(211)
stem(f,abs(C_k))
grid on
xlabel('f')
ylabel('Amplitude Spectrum')
title('Two Sided Spectrum')

subplot(212)
stem(f,angle(C_k)*180/pi)
grid on
```

```
xlabel('f')
ylabel('Phase Spectrum')
```



d) Find the power of the signal.

Using the double-sided spectrum of the Fourier series expansion to find the power. That's using Parseval's theorem:

```
clc
clear all
close all
C_k = zeros(1,11);

C_k(1)  = (-2/1j)*exp(1j*(40)); %C-5
C_k(3)  = -5*exp(-1j*(pi-70)); %C-3
C_k(6)  =2*1j; %C_0
C_k(9)  =-5*exp(1j*(pi-70)) ; %C_3
C_k(11)  = (-2/1j)*exp(1j*(-40)); %C_5
P = abs((C_k(6)))^2 + abs((C_k(1)))^2 +abs((C_k(3)))^2
```

```
Command Window
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P =

   33.0000
```

4- Suppose that a signal $x(t)$ is given by $x(t) = te^{-3t}\,u(t)$. Compute the Fourier Transform $X(\omega)$ of the signal and plot for X-axis is $-20 \le \omega \le 20\ rad/sec$.

Using the Fourier Transform Pair:

$$e^{-at}u(t) \underset{FT}{\leftrightarrow} \frac{1}{a + jw}$$

So,

$$e^{-3t}u(t) \underset{FT}{\leftrightarrow} \frac{1}{3 + jw}$$

Using Multiplication with 't' property of the Fourier Transform:

$$tf(t) \underset{FT}{\leftrightarrow} -\frac{d}{dw}(F(w))$$

So,

$$te^{-3t}u(t) \underset{FT}{\leftrightarrow} -\frac{d}{dw}\left(\frac{1}{3 + jw}\right)$$

$$\boldsymbol{X(jw) = \frac{1}{(3 + jw)^2}}$$

```
%Q3
syms t w
x = t*exp(-3*t)*heaviside(t);
F = fourier(x,w)

F =

1/(3 + w*1i)^2
```

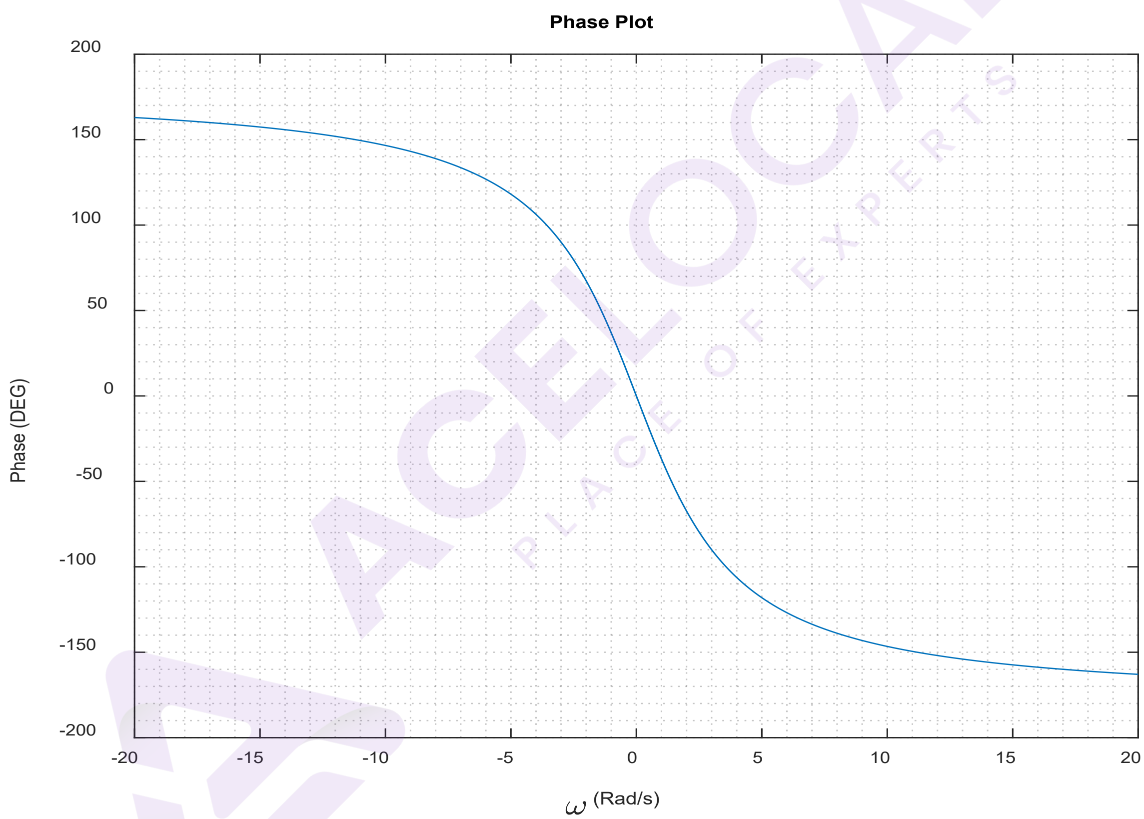
a- The Magnitude of $X(\omega)$

```
ww = -20:0.01:20;
F=subs(F,w,ww);
```

```
plot(ww,abs(F))
grid minor
title('Magnitude Plot')
xlabel('\omega (Rad/s)')
ylabel('Amplitude')
```

b- The Phase of $X(\omega)$

```
figure
plot(ww,angle(F)*180/pi)
grid minor
title('Phase Plot')
xlabel('\omega (Rad/s)')
ylabel('Phase (DEG)')
```



c- Calculate the inverse Fourier transform of $X(\omega)$

```
clear all
close all
clc
syms w t
F = 1/(3 + w*1i)^2;
x = ifourier(F,t);
x=simplify(x);
pretty(x)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
t exp(-3 t) (sign(t) + 1)
--------------------------
            2
```

5- Use Matlab to plot the spectrum of the following two signals: sss

a- $x(t) = \frac{\sin(\pi t)}{\pi t}$

```
clc
clear all
close all

syms t w
x1 = sin(pi*t) / (pi*t);
F1 = fourier(x1,w)
ww = -10:0.01:10;

F1 = subs(F1,w,ww);
plot(ww,F1)
grid minor
title('Magnitude Plot')
xlabel('\omega (Rad/s)')
ylabel('Amplitude')
ylim([0 1.2])
```

Command Window

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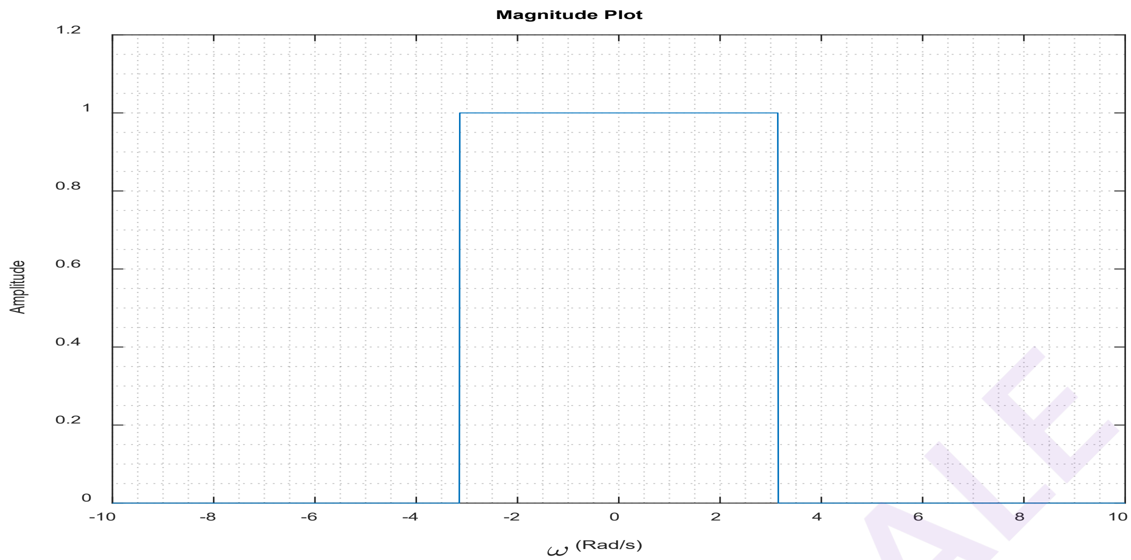
```
F1 =

(pi*heaviside(pi - w) - pi*heaviside(- w - pi))/pi
```



b- $y(t) = \begin{cases} 1 & -1 \leq t \leq 1 \\ 0 & else \end{cases}$

```
clc
clear all
close all

syms t w
x2 = heaviside(t+1) - heaviside(t-1) ;
F2 = fourier(x2,w)
ww = -20:0.03:20;

F2 = subs(F2,w,ww);
plot(ww,F2)
grid minor
title('Magnitude Plot')
xlabel('\omega (Rad/s)')
ylabel('Amplitude')
```

Command Window

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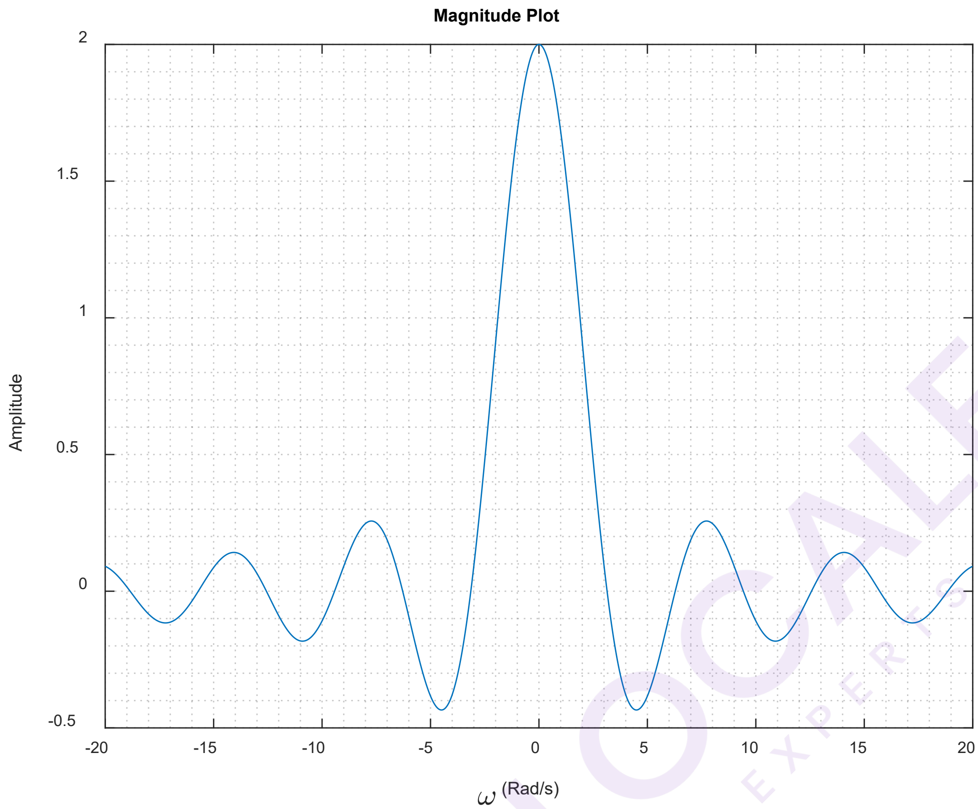
```
F2 =

- (- sin(w) + cos(w)*1i)/w + (sin(w) + cos(w)*1i)/w
```


Magnitude Plot
Amplitude
2
1.5
1
0.5
0
-0.5
-20
-15
-10
-5
0
5
10
15
20
ω (Rad/s)