## Objectives

The main objectives of the experiment are listed below

- This experiment is aimed to prove the law of conservation of mechanical energy by experimental method.
- It is aimed to evaluate the moment of inertia using graphical method.

## Theory

Energy is used for performing any kind of work and a general rule for energy is that it can neither be created nor destroyed but only changes from one form to another. There are numerous forms of energy but one of the most common types is known as mechanical energy. Mechanical energy is further divided into two types of energy which include kinetic energy and potential energy. Kinetic energy corresponds to object motion while potential energy is stored energy relevant to object position. Mathematically, they are given as:

$$K = \frac{1}{2}mv^2 \quad (1)$$

$$U = mgh \quad (2)$$

This lab session focuses on the conservation of energy and the rotational motion of the disk is converted into linear motion of the hanging mass as the energy change takes place. The assembly used for demonstration involves a hanging mass that has gravitational potential energy. The hanging mass is present at a certain height from the ground and upon falling, the height decreases thus converting the potential energy into mass kinetic energy and rotational energy of the platter.

The energy remains conserved but only changes from one form to other.

$$E_p = E_T + E_R \quad (3)$$

## Equipment

For the execution of this lab session following components were employed:

- Platter
- Step pulley
- Spindle
- Photogate
- Smart pulley
- Mass
- Mass holder

The visual representation of the assembly is given as shown below:

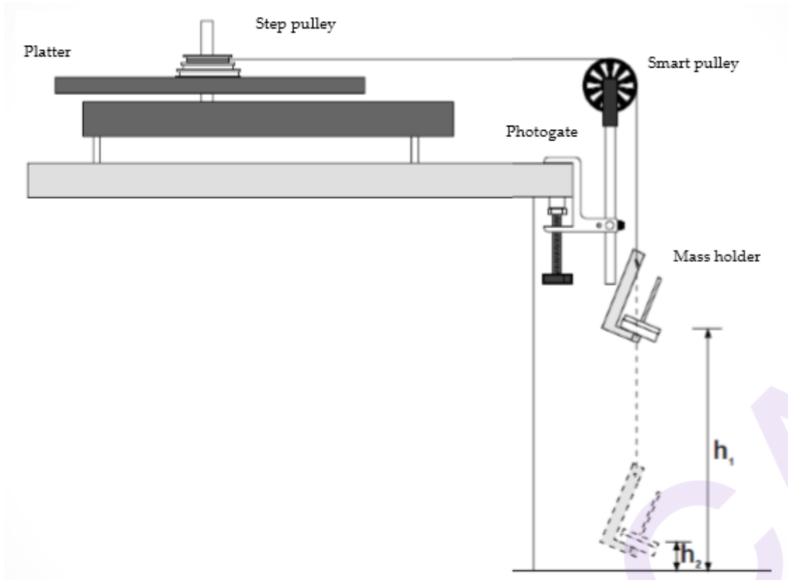


Figure 1 Apparatus Diagram

## Procedure

The following steps were followed for the execution of the experiment:

- The apparatus was set and all components were connected. The connections were checked.
- The mass holder was set and it was raised to the required height h and it was measured.
- The initial mass of 60 units was added and it was allowed to fall.
- The corresponding values of speed and acceleration were measured, evaluated, and tabulated.
- The mass values were increased in form of increments to 80, 100, 120, and 140. The corresponding values of speed and acceleration were measured and tabulated.
- For all values of masses, three trials were performed and the values of required parameters were noted and tabulated.
- The collected data was used for further calculations and analysis.

## Data

Table 1 Recorded data

| **Mass** (kg) | **Average Speed (m/sec)** | **Average Acceleration (m/sec$^2$)** |
|---|---|---|
| 0.03 | 0.11 | 0.01 |
| 0.06 | 0.17 | 0.03 |
| 0.08 | 0.2 | 0.04 |
| 0.10 | 0.22 | 0.05 |
| 0.12 | 0.26 | 0.06 |

## Analysis

Table 2 Part I Analysis Data Table

| Mass | $a_{av}$ | $u(a_{av})$ | $mr^2(g-a_{av})$ | $u(mr^2(g-a_{av}))$ |
|---|---|---|---|---|
| (kg) | (m/s$^2$) | (m/s$^2$) | Kgm$^3$/s$^2$ | Kgm$^3$/s$^2$ |
| 0.03 | 0.01 | 0.0001 | 0.000118363 | 2.94802E-05 |
| 0.06 | 0.03 | 0.0001 | 0.000236242 | 5.88399E-05 |
| 0.08 | 0.04 | 0.0001 | 0.000314667 | 7.83729E-05 |
| 0.10 | 0.05 | 0.0001 | 0.000392930 | 9.78658E-05 |
| 0.12 | 0.06 | 0.0001 | 0.000471033 | 0.000117319 |

Table 3 Part II Analysis data table

| Mass | $v_{av}$ | $u(v_{av})$ | mgh | u(mgh) | $1/2mv_{av}^2$ | $U(1/2mv_{av}^2)$ | $1/2I\omega^2_{av}$ | $U(1/2I\omega^2_{av})$ |
|---|---|---|---|---|---|---|---|---|
| (kg) | (m/s) | (m/s) | (J) | (J) | (J) | (J) | (J) | (J) |
| 0.03 | 0.11 | 0.01 | 0.18816 | 0.06279 | 0.0002 | 6.27096E-05 | 0.120879 | 0.0614 |
| 0.06 | 0.17 | 0.01 | 0.37632 | 0.06300 | 0.0009 | 0.000153236 | 0.288711 | 0.1454 |
| 0.08 | 0.2 | 0.01 | 0.50176 | 0.06321 | 0.0016 | 0.000215407 | 0.3996 | 0.2009 |
| 0.10 | 0.22 | 0.01 | 0.6272 | 0.06348 | 0.0024 | 0.000265827 | 0.483516 | 0.2428 |
| 0.12 | 0.26 | 0.01 | 0.75264 | 0.06382 | 0.0041 | 0.000372263 | 0.675324 | 0.3388 |

## Graphs

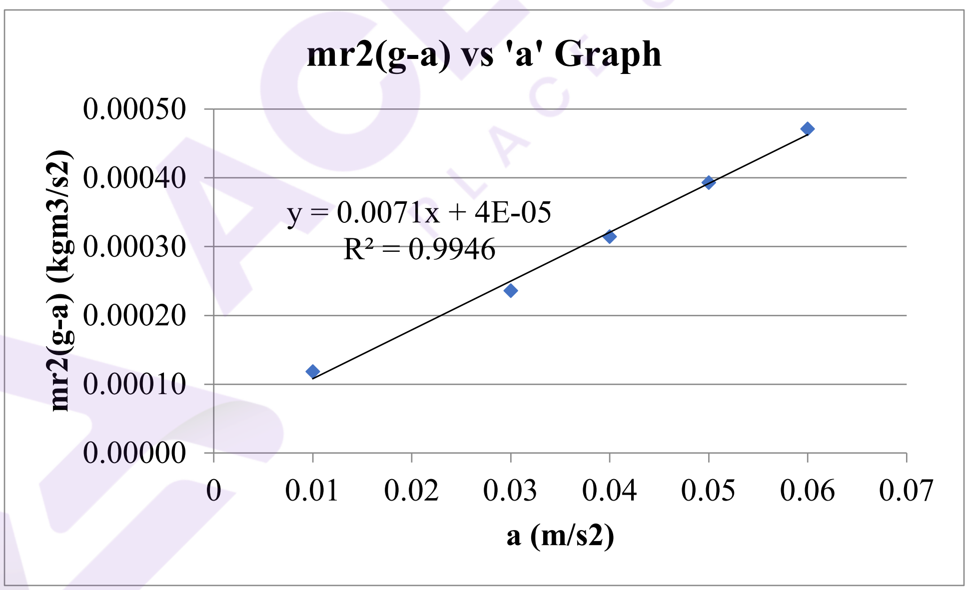


Figure 2 $mr^2(g-a)$ vs 'a' Graph

Table 4 Uncertainty in slope and intercept using LINEST function

| | | | |
|---|---|---|---|
| Slope | 0.00710 | Intercept | 0.000040 |
| Uncertainty in Slope | 0.00004 | Uncertainty in intercept | 0.000001 |

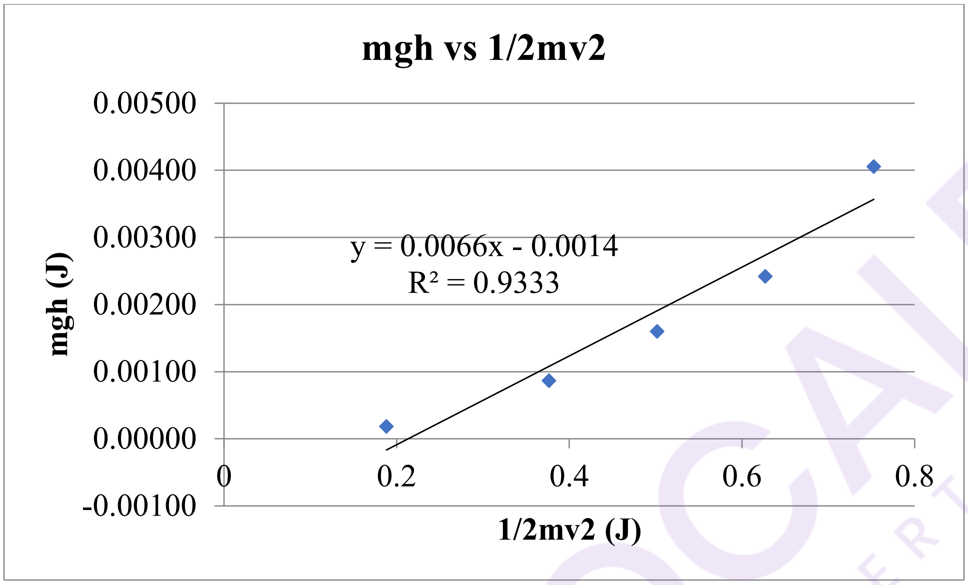


Figure 3 Potential energy as a function of kinetic energy

Table 5 Uncertainty in slope and intercept using LINEST function

| | | | |
|---|---|---|---|
| Slope | 0.00660 | Intercept | 0.0014 |
| Uncertainty in Slope | 0.00001 | Uncertainty in intercept | 0.0001 |

## Results

$$I_{th} = \frac{1}{2}MR^2$$

$$I_{th} = \frac{1}{2} \times 0.991 \times 0.127^2 = 0.00799\ Kgm^2$$

$$I_{exp} = slope = 0.0071$$

The error percentage between the theoretical and experimental value is given by:

$$\%error = \frac{Theoretical\ value - Experimental\ value}{Theoretical\ value} \times 100\%$$

$$\text{Percentage error} = \left|\frac{0.00799 - 0.0071}{0.00799}\right| = 10\%$$

Error Analysis

- The error in the mass (m) is 0.001 Kg.
- The error in the height (h) is 0.01 m.
- The error in the radius (R) is 0.001 m.
- The error on velocity is obtained by using standard deviation of the average method as follows.

$$\sigma_{vmean} = \sqrt{\frac{\sum(v_i - v_{av})^2}{N-1}}$$

$$\sigma_{vmean} = \sqrt{\frac{(v_1 - v_{av})^2 + (v_2 - v_{av})^2 + (v_3 - v_{av})^2}{n-1}}$$

$$\sigma_{vmean} = 0.001\ m/sec$$

- The error on acceleration is obtained by using standard deviation of the average method as follows.

$$\sigma_{amean} = \sqrt{\frac{\sum(a_i - a_{av})^2}{N-1}}$$

$$\sigma_{amean} = \sqrt{\frac{(a_1 - a_{av})^2 + (a_2 - a_{av})^2 + (a_3 - a_{av})^2}{n-1}}$$

$$\sigma_{amean} = 0.001 m/s^2$$

a. Estimation of the error on $\Delta I_{th}$

$$\Delta I_{th} = \Delta\left(\frac{1}{2}MR^2\right) = \sqrt{(\frac{1}{2}.R^2.\Delta M)^2 + (M.R.\Delta R)^2}$$

$$\Delta I_{th} = 0.00005\ kgm^2$$

$$\Delta I_{exp} = \Delta(slope)$$

$$\Delta I_{exp} = 0.00001\ kgm^2$$

## Discussion & Conclusion

In this experiment, analysis is carried out in two different parts. In the part I, acceleration is measured by using which moment of inertia is calculated using the linear line equation. Theoretical value of moment of inertia is also calculated and percentage error is calculated to be 10 % between experimental and theoretical values. Theoretical values and experimental values of I are found to be 0.00799 ± 0.00005 $kgm^2$ and 0.0071 ± 0.00001 $kgm^2$ respectively. The difference in experimental and theoretical values is due to instrumental, human or random errors. The quantitative error analysis is carried out and uncertainty in each parameter is reported.

In the second part, potential energy and kinetic energies are calculated and graph of potential energy as a function of kinetic energy is obtained. The slope of the graph is calculated and its uncertainty is reported using the LINEST function. It was concluded that decrease in potential energy causes an increase in kinetic energy thus conserving the overall mechanical energy of the system.

The conservation of mechanical energy was thoroughly studied and discussed along with its components. The values of velocities, accelerations, potential energies, kinetic energies, and rotational energies for corresponding masses were measured and tabulated. Using the collected data, the conservation of mechanical energy was validated. The modified linear equations were formed and slope and intercept were found and graphs were also plotted. The error analysis was performed and errors along with their sources were identified.

## References


1. Bryan, J., 2004. Video analysis software and the investigation of the conservation of mechanical energy. *Contemporary Issues in Technology and Teacher Education*, *4*(3), pp.284-298.
2. Maloney, D.P., 1985. Rule-governed approaches to physics: Conservation of mechanical energy. *Journal of Research in Science Teaching*, *22*(3), pp.261-278.
3. Bryan, J.A., 2010. Investigating the conservation of mechanical energy using video analysis: four cases. *Physics Education*, *45*(1), p.50.