Q:1

$$\frac{dy}{dx} + p(x)y = f(x)y$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(f(x) - p(x)\right)y$$

Since "x" and "y" variables can be Separated, So this is Separable.

Now

$$\frac{dy}{dx} + \left(p(x) - f(x)\right)y = 0$$

This is Linear ODE as well.

Now $\quad \frac{dy}{dx} + p(x)y = f(x)y$

$$\hookrightarrow \left[(p(x) - f(x))y\right]dx + dy = 0$$

Here $\quad M(x,y) = \left(p(x) - f(x)\right)y$

$$\hookrightarrow \frac{\partial M}{\partial y} = p(x) - f(x)$$

and $\quad N(x,y) = 1 \quad \Rightarrow \quad \frac{\partial N}{\partial x} = 0$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ So Given DE is not EXACT

So Given Statement is

$$\boxed{\text{TRUE}}$$

Q:2 $\quad e^{z-1} = -ie^2$

$\Rightarrow e^z \cdot e^{-1} = -ie^2$

$\hookrightarrow \cancel{e}\ e^z = -ie^3$

Now Taking Natural Logarith on both sides:

$$\ln(e^z) = \ln(-ie^3)$$

$\hookrightarrow z\ln(e) = \ln(-i) + \ln(e^3)$

$\Rightarrow z(1) = \ln(-i) + 3\ln(e)$

$\Rightarrow z = 3 + \ln(-i) \longrightarrow$ (i)

Now

$$\ln(-i) = \log_e |i| + i\arg(-i) \longrightarrow \text{(ii)}$$

Now $\quad |i| = \sqrt{(0)^2+(1)^2} = \sqrt{1} \Rightarrow |i| = 1$

and $\arg(-i) = \tan^{-1}\left(\frac{-1}{0}\right)$ as $x=0,\ y=1$

So $\quad \arg(-i) = \frac{3\pi}{2} + 2n\pi$

So eq (ii) $\Rightarrow \ln(-i) = \log_e(1) + i\left(\frac{3\pi}{2} + 2n\pi\right)$

$\hookrightarrow \ln(-i) = 0 + \left(\frac{3\pi}{2} + 2n\pi\right)i$

So

eq(i) $\Rightarrow$ $\boxed{z = 3 + \left(\frac{3\pi}{2} + 2n\pi\right)i}$

Q:3 $(y^2\cos y - x)\,dy + y\,dx = 0$

$M(x,y) = y \Rightarrow \frac{\partial M}{\partial y} = 1$

$N(x,y) = y^2\cos y - x \Rightarrow \frac{\partial N}{\partial x} = -1$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, So DE is not exact.

Now

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-1-1}{y} = \frac{-2}{y}$$

So Special integrating Factor is

$$\mu(y) = e^{\int\left(\frac{N_x - M_y}{M}\right)dy}$$

$$\mu(y) = e^{\int \frac{-2}{y}dy} = e^{-2\ln|y|}$$

$$\mu(y) = e^{\ln(y^{-2})}$$

$$\boxed{\mu(y) = y^{-2}}$$

Q:4 $$\frac{dy}{dx} = \frac{-2y^2-2y-4x^2}{2xy+x} \longrightarrow (i)$$

Let $u = -2y^2-2y-4x^2$

$$\hookrightarrow \frac{du}{dx} = -4y\frac{dy}{dx} - 2\frac{dy}{dx} - 8x$$

$$\Rightarrow \frac{du}{dx} + 8x = -2(2y+1)\frac{dy}{dx}$$

$$\hookrightarrow -\frac{1}{2}\frac{du}{dx} - 4x = (2y+1)\frac{dy}{dx} \longrightarrow (ii)$$

So

eq (i) $\Rightarrow$ $$\frac{dy}{dx} = \frac{-2y^2-2y-4x^2}{x(2y+1)}$$

$$\hookrightarrow (2y+1)\frac{dy}{dx} = \frac{-2y^2-2y-4x^2}{x} \longrightarrow (iii)$$

Comparing equations (ii) and (iii)

$$-\frac{1}{2}\frac{du}{dx} - 4x = \frac{-2y^2-2y-4x^2}{x}$$

$$\hookrightarrow -\frac{1}{2}\frac{du}{dx} - 4x = \frac{u}{x} \qquad (\because u = -2y^2-2y-4x^2)$$

$$\Rightarrow -\frac{1}{2}\frac{du}{dx} - \frac{u}{x} = 4x$$

$$\hookrightarrow \frac{du}{dx} + \frac{2}{x}u = -8x \longrightarrow (iv)$$

$\hookrightarrow$ Linear Eq.

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Q:4 Remaining

integrating factor is $\mu(x) = e^{\int \frac{2}{x} dx}$

$\hookrightarrow \mu(x) = e^{2\ln|x|} = e^{\ln(x^2)} = x^2$

So Multiplying eq (iv) with "x^2"

$$x^2 \frac{du}{dx} + 2xu = -8x^3$$

$$\Rightarrow \frac{d}{dx}(x^2 u) = -8x^3 \Rightarrow x^2 u = \int -8x^3 \, dx$$

$$\Rightarrow x^2 u = -2x^4 + C_1$$

As $u = -2y^2 - 2y - 4x^2$

So

$$x^2(-2y^2 - 2y - 4x^2) = -2x^4 + C_1$$

$$-2x^2y^2 - 2x^2y - 4x^4 = -2x^4 + C_1$$

$$\Rightarrow -2x^2y^2 - 2x^2y - 4x^4 + 2x^4 = C_1$$

Dividing by -2 $\Rightarrow$ $\boxed{x^2y^2 + x^2y + x^4 = C}$

where $C = \frac{C_1}{-2}$

So OPTION (A)

Q:5 $\frac{dy}{dx} = 6x e^{6x+3y} + \sin(6x) e^{3y}$

$\hookrightarrow \frac{dy}{dx} = (e^{3y})(6xe^{6x} + \sin(6x))$

Separating variables and integrating

$$\int e^{-3y} dy = \int (6x e^{6x} + \sin(6x)) dx$$

$$\Rightarrow \frac{e^{-3y}}{-3} = (6)\int x e^{6x} dx + \int \sin(6x) dx$$

$$\Rightarrow -\frac{e^{-3y}}{+3} = 6\left[(x)\left(\frac{e^{6x}}{6}\right) - \int \frac{e^{6x}}{6} dx\right] + \left(-\frac{\cos(6x)}{6}\right)$$

$$\Rightarrow -\frac{1}{3} e^{-3y} = x e^{6x} - \int e^{6x} dx - \frac{1}{6}\cos(6x)$$

$$\Rightarrow -\frac{1}{3} e^{-3y} = x e^{6x} - \frac{e^{6x}}{6} - \frac{1}{6}\cos(6x) + C_1$$

$$\Rightarrow e^{-3y} = -3x e^{6x} + \frac{1}{2} e^{6x} + \frac{1}{2}\cos(6x) + C$$

$C = -3C_1$

Using $y(0) = 0$

$\hookrightarrow e^0 = -3(0)e^0 + \frac{1}{2} e^0 + \frac{1}{2}\cos(0) + C$

$\Rightarrow 1 = 0 + \frac{1}{2} + \frac{1}{2} + C \Rightarrow 1 = 1 + C$

$\hookrightarrow C = 0$

So

$$\boxed{e^{-3y} = -3x e^{6x} + \frac{1}{2} e^{6x} + \frac{1}{2}\cos(6x)}$$

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Q:6 We need to solve $\cosh(2+3i)$

We know;

$$\cosh z = \frac{e^z + e^{-z}}{2}$$

We have to put $z = 2+3i$

$$\hookrightarrow \cosh(2+3i) = \frac{e^{2+3i} + e^{-(2+3i)}}{2}$$

$$\hookrightarrow \cosh(2+3i) = \frac{e^2 e^{3i} + e^{-2} e^{-3i}}{2}$$

We know $e^{i\theta} = \cos\theta + i\sin\theta$

So

$$\cosh(2+3i) = \frac{(e^2)\big(\cos(3) + i\sin(3)\big) + e^{-2}\big(\cos(-3) + i\sin(-3)\big)}{2}$$

$$\hookrightarrow \cosh(2+3i) = \frac{e^2(-0.9900 + 0.1411i) + e^{-2}(-0.9900 - 0.1411i)}{2}$$

$$\Rightarrow \cosh(2+3i) = \frac{-7.3151 + 1.0427i - 0.1340 - 0.0191i}{2}$$

$$\hookrightarrow \cosh(2+3i) = \frac{-7.4491}{2} + \frac{1.0236}{2} i$$

$$\hookrightarrow \boxed{\cosh(2+3i) = -3.7245 + 0.5118i}$$

Q:7 $\left(e^{x^2} - y\cos(xy)\right)dx + \left(e^{y} - x\cos(xy)\right)dy = 0$

Here $M = e^{x^2} - y\cos(xy)$

$\hookrightarrow M_y = \frac{\partial}{\partial y}\left(e^{x^2} - y\cos(xy)\right)$

$\hookrightarrow M_y = 0 - \left[(y)\frac{\partial}{\partial y}(\cos(xy)) + (\cos(xy))\frac{\partial}{\partial y}(y)\right]$

$\Rightarrow M_y = (-y)(-\sin(xy))(x) - (\cos(xy))(1)$

$\Rightarrow M_y = xy\sin(xy) - \cos(xy) \longrightarrow (i)$

Now $N = e^{y} - x\cos(xy)$

$\hookrightarrow N_x = \frac{\partial}{\partial x}\left(e^{y} - x\cos(xy)\right)$

$\Rightarrow N_x = 0 - \left[(x)\frac{\partial}{\partial x}(\cos(xy)) + (\cos(xy))\frac{\partial}{\partial x}(x)\right]$

$\Rightarrow N_x = -x(-\sin(xy))(y) - (\cos(xy))(1)$

$\hookrightarrow N_x = xy\sin(xy) - \cos(xy) \longrightarrow (ii)$

From eq (i) and (ii)

Since $M_y = N_x$

So $\boxed{\text{DE is EXACT}}$

Q:8 $\frac{dy}{dx} e^{x} y = e^{-y} + e^{-2x-y}$

$\Rightarrow \frac{dy}{dx} e^{x} y = e^{-y} + e^{-2x} e^{-y}$

$\hookrightarrow \frac{dy}{dx} \cdot e^{x} y = e^{-y}\left(1+e^{-2x}\right)$

$\hookrightarrow \frac{dy}{dx} = \frac{e^{-y}\left(1+e^{-2x}\right)}{e^{x} y}$

$\hookrightarrow \frac{dy}{dx} = \left(\frac{e^{-y}}{y}\right)\left(\frac{1+e^{-2x}}{e^{x}}\right)$

Since $\frac{dy}{dx}$ can be written in the form of $\frac{dy}{dx} = P(x)\,Q(y)$

So Given DE is

SEPARABLE

Q: 9

$$f(z) = 7z - 9iz - 3 + 2i$$

$$\hookrightarrow f(z) = (7z - 3) + (2 - 9z)i$$

Let $z = x + iy$

$$\hookrightarrow f(z) = 7(x + iy) - 9i(x + iy) - 3 + 2i$$

$$\Rightarrow f(z) = 7x + 7yi - 9xi - 9yi^2 - 3 + 2i$$

$$\Rightarrow f(z) = 7x + 7yi - 9xi + 9y - 3 + 2i \quad (\because i^2 = -1)$$

$$\Rightarrow \boxed{f(z) = (7x + 9y - 3) + (7y - 9x + 2)i}$$

Q:10 $$\frac{dy}{dx} = -\frac{aye^{axy}+2bxe^{bx^2}}{axe^{axy}+2bye^{by^2}}$$

$$\Rightarrow \left(axe^{axy}+2bye^{by^2}\right)dy = -\left(aye^{axy}+2bxe^{bx^2}\right)dx$$

$$\Rightarrow \left(aye^{axy}+2bxe^{bx^2}\right)dx + \left(axe^{axy}+2bye^{by^2}\right)dy = 0$$

$$M = aye^{axy}+2bxe^{bx^2}$$

$$\hookrightarrow M_y = \frac{\partial}{\partial y}\left(aye^{axy}+2bxe^{bx^2}\right)$$

$$\hookrightarrow M_y = (a)\left[(1)e^{axy}+(y)(e^{axy})(ax)\right]+0$$

$$\Rightarrow \quad M_y = ae^{axy}+a^2xye^{axy} \longrightarrow (i)$$

And

$$N = axe^{axy}+2bye^{by^2}$$

$$\hookrightarrow N_x = \frac{\partial}{\partial x}\left(axe^{axy}+2bye^{by^2}\right)$$

$$\hookrightarrow N_x = (a)\left[(1)e^{axy}+(x)(e^{axy})(ay)\right]+0$$

$$\hookrightarrow N_x = ae^{axy}+a^2xye^{axy} \longrightarrow (ii)$$

Since $M_y = N_x$ So given DE is exact. So there is a function $F(x,y)$ Such that

$$F_x = M = aye^{axy}+2bxe^{bx^2} \longrightarrow (iii)$$

$$F_y = N = axe^{axy}+2bye^{by^2} \longrightarrow (iv)$$

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Q:10 Remaining

Integrating eq (iii) w.r.t "x"

$$F(x,y) = \int \left(aye^{axy} + 2bxe^{bx^2}\right)dx + g(y)$$

$$\hookrightarrow F(x,y) = e^{axy} + e^{bx^2} + g(y) \longrightarrow (v)$$

Differentiating eq (v) w.r.t "y" Keeping "x" constant

$$F_y = axe^{axy} + g'(y) \longrightarrow (vi)$$

Comparing eq (iv) and (vi)

$$g'(y) = 2bye^{by^2}$$

$$\hookrightarrow g(y) = \int \left(2bye^{by^2}\right)dy$$

$$\hookrightarrow g(y) = e^{by^2} + C$$

So

$$F(x,y) = e^{axy} + e^{bx^2} + e^{by^2} + C$$

An implicit solution is

$$\boxed{e^{bx^2} + e^{by^2} + e^{axy} = \text{Constant}}$$