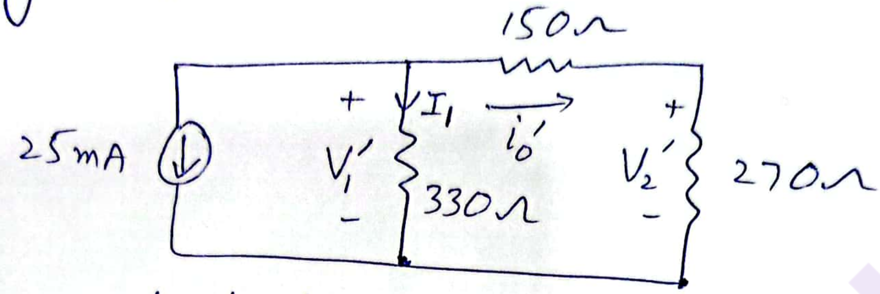
Problem set # 2

Q#01

Using superposition



Using current divider Rule

$$i_o' = -\frac{330}{330+150+270}(25mA)$$

$$i_o' = -11mA$$

Using Ohm's Law:

$$V_2' = i_o' \times 270$$

$$V_2' = -11 \times 10^{-3} \times 270$$

$$V_2' = -2.97\ V$$

Using KCL

$$25mA + I_1 + i_o' = 0$$

$$\Rightarrow \quad I_1 = -25mA - i_o'$$

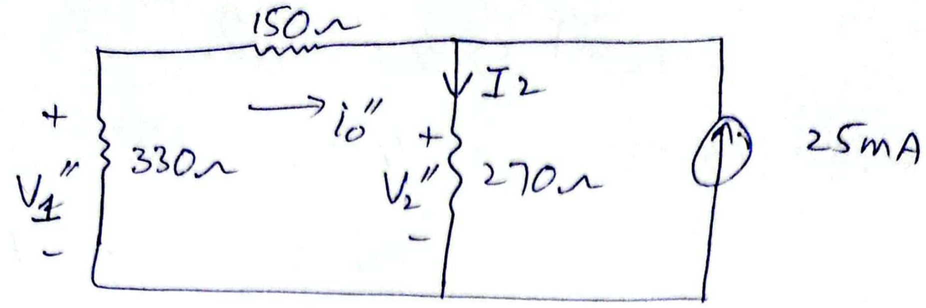
$$I_1 = -25mA - (-11mA)$$

$$I_1 = -25mA + 11mA = -14mA$$

Using Ohm's Law:

$$V_1' = 330\, I_1 = 330 \times (-14 \times 10^{-3})$$

$$V_1' = -4.62\ V$$



Using current divider

$$i_o'' = \frac{-270}{270+150+330}(25mA)$$

$$i_o'' = -9mA$$

Using Ohm's Law:

$$V_1'' = -330\, i_o'' = -330(-9\times10^{-3})$$

$$V_1'' = 2.97\,V$$

Using KCL

$$i_o'' + 25mA = I_2$$

$$\Rightarrow \quad I_2 = -9mA + 25mA$$

$$\Rightarrow \quad I_2 = 16mA$$

Using Ohm's Law:

$$V_2'' = 270\, I_2 = 270\times16\times10^{-3}$$

$$V_2'' = 4.32\,V$$

Now, using superposition

$$i_o = i_o' + i_o'' = -11mA - 9mA = \boxed{-20\,mA}$$

$$V_1 = V_1' + V_1'' = -4.62 + 2.97 = \boxed{-1.65\,V}$$

$$V_2 = V_2' + V_2'' = -2.97 + 4.32 = \boxed{1.35\,V}$$

Q#02

a) $\because \; V_s(t) = 30\cos(12990t + 30^\circ)\ V$

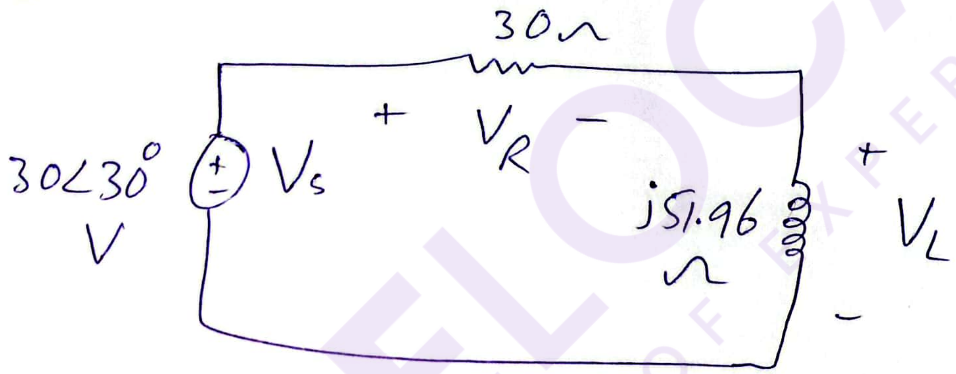
So, $\boxed{\omega = 12990 \ Rad/s}$

b) In phasor domain

$$V_s = 30\angle 30^\circ \ V$$

$$X_L = j\omega L = j \times 12990 \times 4 \times 10^{-3}$$

$$X_L = j51.96\,\Omega$$



c)

Using voltage divider

$$V_L = \frac{j51.96}{30 + j51.96}(30\angle 30^\circ) = 25.98\angle 60^\circ \ V$$

$$V_R = \frac{30}{30 + j51.96}(30\angle 30^\circ) = 15\angle -30^\circ \ V$$

Now, in Time domain

$$\boxed{V_L(t) = 25.98\cos(12990t + 60^\circ)\ V}$$

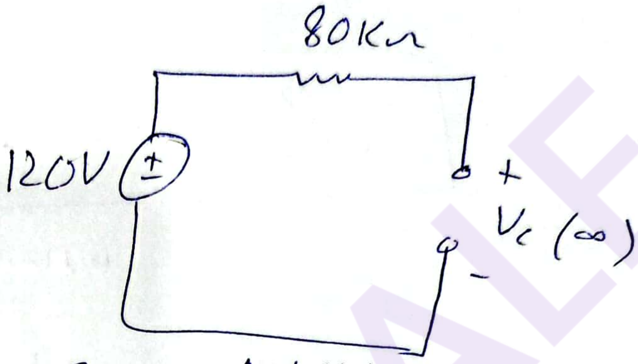
$$\boxed{V_R(t) = 15\cos(12990t - 30^\circ)\ V}$$

Q#03

(a) For $t<0$ switch open,

So, $V_c(0^-) = 0\,V$

For $t \to \infty$



$$V_c(\infty) = 120V$$

For $t \geq 0$

$$\tau = RC = 80\times10^{3}\times 25\times10^{-9}$$

$$\tau = \frac{1}{500}\ s$$

Now,

$$V_c(t) = V_c(\infty) + \{V_c(0^+) - V_c(\infty)\}\, e^{-t/\tau}$$

$$V_c(t) = 120 + \{0 - 120\}\, e^{-t\,500}$$

$$\boxed{V_c(t) = 120\left(1 - e^{-500t}\right)\ V,\quad t \geq 0}$$

(b) $\because\ i(t) = C\dfrac{dV_c}{dt}$

$$i(t) = 25\times10^{-9}\times\frac{d}{dt}\left(1 - e^{-500t}\right)$$

$$i(t) = 25\times10^{-9}\left(500\, e^{-500t}\right)$$

$$\boxed{i(t) = 12.5\, e^{-500t}\ \mu A,\quad t \geq 0}$$

(c)

$$p(t) = V_c(t)\; i(t)$$

$$p(t) = 120(1-e^{-500t}) \times (12.5\times10^{-6}\, e^{-500t})$$

$$p(t) = (120-120e^{-500t})(12.5\times10^{-6}\, e^{-500t})$$

$$p(t) = (1.5-1.5e^{-500t})\, e^{-500t} \text{ mW}$$

$$\boxed{p(t) = 1.5e^{-500t} - 1.5e^{-1000t} \text{ mW},} \quad t \geq 0$$

(d) $\because w(t) = \int_0^t p(t)\, dt$

$$w(t) = 1.5\int_0^t e^{-500t}\, dt - 1.5\int_0^t e^{-1000t}$$

$$w(t) = \left[\frac{1.5}{-500} e^{-500t}\right]_0^t - \frac{1.5}{-1000}\left[e^{-1000t}\right]_0^t$$

$$w(t) = -0.003\left[e^{-500t}-1\right] + 0.0015\left[e^{-1000t}-1\right]$$

$$w(t) = 0.003 - 0.003e^{-500t} - 0.0015 + 0.0015e^{-1000t}$$

$$\Rightarrow \boxed{w(t) = 1.5 - 3e^{-500t} + 1.5e^{-1000t} \;\; \mu J, \quad t \geq 0}$$